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A HAZARDOUS INSPECTION MODEL

BY

DAVID A. BUTLER

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DEPARTMENT OF OPERATIONS RESEARCH
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STANFORD   UNIVERSITY
STANFORD, CALIFORNIA
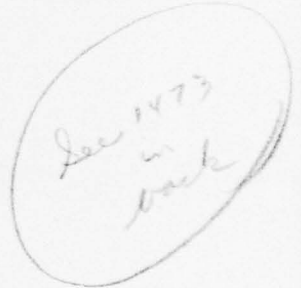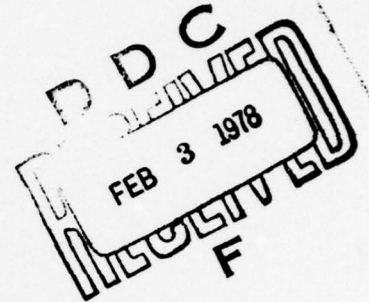
1. Introduction

In this paper we consider a hazardous inspection model. We are
given a device which operates throughout a number of periods and which
in each period is subject to failure. Failure of the device is directly
observable. Prior to failure the device will enter a state in which it
is functioning, but in an impaired manner. This state can be detected
only by performing an inspection. Once the machine is known to be in the
impaired state, appropriate action may be taken to prolong the remaining
life of the device. In this respect inspection is valuable. However,
the act of inspecting the device when it is not impaired may itself
cause the device to become impaired. In this respect inspection is
hazardous. This paper deals with the determination of inspection policies
which maximize the expected lifetime of the device.

The above model is very appropriate for situations where the cost
of inspection is negligable relative to the cost of failure. One important
example is the use of X-rays to detect cancer. Currently there is con-
siderable controversy as to whether X-rays may themselves cause cancer and,
granting at least their potential for causing cancer, whether or not
they should be employed. (See [3] for a detailed list of references on
this subject.) Another prime example is the inspection of nuclear reactors.
Since the largest single cause of malfunctions is human error [11], a
fundamental question is do human inspections create more problems than
they solve.

A number of authors have studied inspection models [1], [5], [6],
[7], [8]. These authors have all assumed a cost structure for inspection
and repair, and have sought to derive inspection and repair policies which

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minimize the overall cost of operation. In none of these papers was
inspection assumed to have any effect (good or bad) on the true state of
the device. Wattanapanom and Shaw [10] consider a hazardous inspection
model in which the device has an exponential failure distribution in the
absence of any inspection. Each inspection either causes immediate failure
or else increases the failure rate. They derive algorithms for finding
inspection policies which minimize the overall operating cost.

2. The Model

Consider a device whose operation can be classified into one of
three categories: fully functional; functional, but impaired; and failed.
The failed state is directly observable but one can distinguish the partially
functional state from the fully functional state only by performing an
inspection. Inspection is perfect (i.e., the true state is always revealed)
and instantaneous.

The inspection model will be formulated as a Markov decision process
[4]. We let X_n denote the true state of the device at the start of
period n (before inspection), where the possible states are

True State	Description
1	fully functional (OK)
2	undetected partial failure (UP)
3	detected partial failure (DP)
4	failed (F) .

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Let a_n denote the action taken at time n, where the possible actions are

Action	Description
0	do not inspect
1	inspect .

We make the usual Markov assumption that the future is in some sense independent of the past. Specifically we assume that

$$\Pr\{X_{n+1} = j \mid X_k, \, a_k, \, k = 1, \, 2, \, \ldots, \, n\}$$

$$= \Pr\{X_{n+1} = j \mid X_n, \, a_n\} \, , \qquad j = 1, \, \ldots, \, 4 \, .$$

Define

$$Q_{ij}(a) = \Pr\{X_{n+1} = j \mid X_n = i, \, a_n = a\} \, , \qquad a = 0, \, 1; \, j = 1, \ldots, 4 \, .$$

We take the one-step transition matrices $Q(0)$ and $Q(1)$ to be as follows.

$Q(0)$
(do not inspect)

	(OK)	(UP)	(DP)	(F)
(OK)	$1-\alpha_0$	α_0	0	0
(UP)	0	$1-\beta$	0	β
(DP)	0	0	$1-\gamma$	γ
(F)	0	0	0	1

$Q(1)$
(inspect)

	(OK)	(UP)	(DP)	(F)
(OK)	$1-\alpha_1$	α_1	0	0
(UP)	0	0	$1-\beta$	β
(DP)	0	0	$1-\gamma$	γ
(F)	0	0	0	1

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We assume that $0 < \alpha_0, \alpha_1, \beta, \gamma < 1$ and that $\alpha_1 > \alpha_0$ and $\beta > \gamma$ (see below).

Note the following features about the model.

1. Once a device enters a state, it never returns to a lower-numbered state.

2. Once a device leaves state 1, the transition probabilities are the same regardless of whether or not it is inspected.

3. Once the device enters state 3 or 4, the decision process is completed, because the future evolution of the device is independent of the actions taken. (This is also true once the device enters state 2; however, state 2 is indistinguishable from state 1 to an observer.)

4. The assumption that $\alpha_1 > \alpha_0$ means that inspection is more hazardous to a fully functional device than is no inspection.

5. The assumption that $\beta > \gamma$ means that once partial failure has been detected, measures can be taken which reduce the periodic probability of failure of the device.

The information available to an observer is whether or not the system is in one of the states $\{1, 2\}$, or state 3, or state 4, and if in state 1 or 2, how many periods have elapsed since the last inspection. [Note that the last inspection must necessarily have indicated the device to be OK.] Thus we define the following observed states.

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Observed State	Description
-1	device failed
0	detected partial failure
$j \geq 1$	device not failed; last inspection OK; last inspection j periods ago

The observed state transitions also have the Markov property. We let Z_n denote the observed state of the device at period n and

$$P_{ij}(a) = \Pr\{Z_{n+1} = j \mid Z_n = i, a_n = a\} , \qquad a = 0, 1; \ i,j \geq -1 .$$

Also let

$$K_j = \Pr\{X_{n+j} = 2 \mid X_n = 1, a_n = 1, a_{n+k} = 0, 0 < k < j\} ,$$

$$L_j = \Pr\{X_{n+j} = 1 \mid X_n = 1, a_n = 1, a_{n+k} = 0, 0 < k < j\} , \quad j = 1, 2, \dots$$

and

$$N_j = K_j + L_j .$$

Note that K_j/N_j is the probability that the true state is 2 (UP) given the observed state is j and L_j/N_j is the probability that the true state is 1 (OK) given the observed state is j.

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Proposition 1.

A. do not inspect

i) $\quad P_{-1,j}(0) = \begin{cases} 1, & j = -1 \\ 0, & \text{otherwise} \end{cases}$

ii) $\quad P_{0,j}(0) = \begin{cases} \gamma, & j = -1 \\ 1-\gamma, & j = 0 \\ 0, & \text{otherwise} \end{cases}$

iii) $\quad P_{i,j}(0) = \begin{cases} K_i\beta/N_i, & j = -1 \\ (L_i+K_i(1-\beta))/N_i, & j = i+1, \quad i \geq 1 \\ 0, & \text{otherwise.} \end{cases}$

B. inspect

i) $\quad P_{-1,j}(1) = \begin{cases} 1, & j = -1 \\ 0, & \text{otherwise} \end{cases}$

ii) $\quad P_{0,j}(1) = \begin{cases} \gamma, & j = -1 \\ 1-\gamma, & j = 0 \\ 0, & \text{otherwise} \end{cases}$

iii) $\quad P_{i,j}(1) = \begin{cases} K_i\beta/N_i, & j = -1 \\ K_i(1-\beta)/N_i, & j = 0 \\ L_i/N_i, & j = 1 \\ 0, & \text{otherwise.} \end{cases} \quad i \geq 1$

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Proof. Parts A.i-ii and B.i-ii are obvious;

$$P_{i,-1}(0) = \Pr\{X_{n+1} = 4, X_n = 2 | Z_n = i\}$$

$$= \Pr\{X_{n+1} = 4 | X_n = 2\} \cdot \Pr\{X_n = 2 | Z_n = i\} = \beta K_j / N_j .$$

Also, it is clear that $P_{ij}(0) = 0$ for all $j \neq -1, i+1$, so that $P_{i,-1}(0) + P_{i,i+1}(0) = 1$. This proves part A.iii. To prove part B.iii note that

$$P_{i,-1}(1) = \Pr\{X_{n+1} = 4, X_n = 2 | Z_n = i\}$$

$$= \Pr\{X_{n+1} = 4 | X_n = 2\} \cdot \Pr\{X_n = 2 | Z_n = i\} = \beta K_i / N_i .$$

Also

$$P_{i,0}(1) = \Pr\{X_{n+1} = 3, X_n = 2 | Z_n = i, a_n = 1\}$$

$$= \Pr\{X_{n+1} = 3 | X_n = 2, a_n = 1\} \cdot \Pr\{X_n = 2 | Z_n = i\}$$

$$= (1-\beta) K_i / N_i .$$

Since $P_{i,j}(1) = 0$ for all other j except $j = 1$, the proof is complete. $\square$

Proposition 2.

A. $L_i = (1-\alpha_1)(1-\alpha_0)^{i-1}$, $\qquad\qquad\qquad\qquad\qquad\qquad i \geq 1.$

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B.

i) $K_i = \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1)(1-\beta)^{i-2} \sum\limits_{k=0}^{i-2} \left(\dfrac{1-\alpha_0}{1-\beta}\right)^k$, $i \geq 1$

ii) $K_i = \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1) \, [(1-\beta)^{i-1} - (1-\alpha_0)^{i-1}]/(\alpha_0-\beta)$,

$$i \geq 1, \; \alpha_0 \neq \beta$$

iii) $K_i = \alpha_1(1-\alpha_0)^{i-1} + (i-1) \, \alpha_0(1-\alpha_1)(1-\alpha_0)^{i-2}$, $i \geq 1, \; \alpha_0 = \beta$.

[Note: By convention, the summation in B.i is zero when $i - 2 < 0$.]

Proof.

A. $L_i = \Pr\{X_{n+k} = 1, \, 0 < k \leq i \,|\, X_n = 1, \, a_n = 1, \, a_{n+k} = 0, \, 0 < k < i\}$

$= (1-\alpha_1)(1-\alpha_0)^{i-1}.$

B. $K_i = \Pr\{X_{n+k} = 2, \, 0 < k \leq i \,|\, X_n = 1, \, a_n = 1, \, a_{n+k} = 0, \, 0 < k < i\}$

$+ \sum\limits_{\ell=2}^{i} \Pr\{X_{n+k} = 1, \, 0 < k < \ell, \, X_{n+j} = 2,$

$\ell \leq j \leq i \,|\, X_n = 1, \, a_n = 1, \, a_{n+k} = 0, \, 0 < k \leq i\}$

$= \alpha_1(1-\beta)^{i-1} + \sum\limits_{\ell=2}^{i} (1-\alpha_1)(1-\alpha_0)^{\ell-2} \cdot \alpha_0(1-\beta)^{i-\ell}$

$= \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1)(1-\beta)^{i-2} \sum\limits_{k=0}^{i-2} \left(\dfrac{1-\alpha_0}{1-\beta}\right)^k$.

This proves (i) from which (iii) follows directly. To prove (ii), note that

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$$K_i = \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1)(1-\beta)^{i-2} \left[\frac{1 - ((1-\alpha_0)/(1-\beta))^{i-1}}{1 - (1-\alpha_0)/(1-\beta)} \right] , \quad \alpha_0 \neq \beta .$$

Multiplying the numerator and denominator of the second term by $(1-\beta)$ yields

$$= \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1) \left[\frac{(1-\beta)^{i-1} - (1-\alpha_0)^{i-1}}{(1-\beta) - (1-\alpha_0)} \right]$$

from which (ii) follows directly. □

To allow for the possibility of alternate failure modes, competing risks and the like, we will assume that every period there is a probability of secondary failure $1-\delta$. We allow the possibility that $\delta = 1$ in order to cancel this feature of the model.

It turns out that the parameter δ acts almost exactly like an ordinary discount factor. Clearly, this aspect of the model could have been incorporated into the transition matrices $Q(\cdot)$ directly. However, our treatment shows the discount-factor-like nature of δ and keeps the transition matrices $Q(\cdot)$ more simple.

We now consider the main objective of this paper, the determination of an inspection policy which yields the maximal expected life of the device. Let $V(s,n)$ denote the maximal expected remaining time until failure of the device given the device currently is in (observed) state s and the device is to be destroyed n periods in the future (if it survives that long). Let $V(s)$ denote the maximal expected remaining time until failure

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of the device given it is currently in (observed) state s. As is rather generally true of dynamic programs, the finite horizon optimal values $V(s,n)$ converge to the infinite horizon optimal values $V(s)$ as n approaches infinity [9]. That is

$$V(s) = \lim_{n \to \infty} V(s,n) , \qquad\qquad s = -1, 0, 1, \ldots \qquad\qquad (1)$$

The recursive relations among the V's are given below.

Proposition 3.

A. Finite Horizon

i) $V(s,0) = 0$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s = -1, 0, 1, \ldots$

ii) $V(-1,n) = 0;\ V(0,n) = (1-(\delta(1-\gamma))^n)/(1-\delta(1-\gamma))$, $\qquad n = 1, 2, \ldots$

iii) $V(s,n) = 1 + \delta \max\{K_s(1-\beta)\ V(0,n-1)/N_s + L_s\ V(1,n-1)/N_s,$
$\qquad\qquad\qquad\qquad (K_s(1-\beta) + L_s)\ V(s+1,n-1)/N_s\}$, $\quad s,n \geq 1$.

B. Infinite Horizon

i) $V(-1) = 0,\ V(0) = 1/(1-\delta(1-\gamma))$,

ii) $V(s) = 1 + \delta \max\{K_s(1-\beta)\ V(0)/N_s + L_s\ V(1)/N_s,$
$\qquad\qquad\qquad\qquad (K_s(1-\beta) + L_s)\ V(s+1)/N_s\}$, $\qquad s \geq 1$.

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Proof.

$$V(0,n) = \sum_{k=0}^{\infty} \Pr\{\text{remaining life of device exceeds } k \,|\, \text{device currently in true state } 3\}$$

$$= \sum_{k=0}^{n-1} (\delta(1-\gamma))^k$$

$$= (1 - (\delta(1-\gamma))^n)/(1 - \delta(1-\gamma)) \quad .$$

The formula for $V(0)$ follows directly from the above by taking the limit as n approaches infinity.

By the so-called principle of optimality of dynamic programming [2],

$$V(s,n) = \max_{a = 0,1} \{1 + \delta \sum_{j=-1}^{\infty} P_{sj}(a) V(j, n-1)\} \quad . \qquad (2)$$

Recalling Proposition 1, the recursive formula for $V(s,n)$ follows immediately. The recursive formula for $V(s)$ follows from taking limits in the above [9]. □

3. Preliminary Analysis

We begin the analysis of the model by investigating the behavior of the quantities K_i/N_i and L_i/N_i.

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<u>Proposition 4.</u>

    i) If $\alpha_0 > \alpha_1\beta$, then K_i/N_i is strictly increasing in i, and L_i/N_i and $(K_i(1-\beta) + L_i)/N_i$ are strictly decreasing in i.

    ii) If $\alpha_0 = \alpha_1\beta$, then K_i/N_i, L_i/N_i, and $(K_i(1-\beta) + L_i)/N_i$ are constant in i.

    iii) If $\alpha_0 < \alpha_1\beta$, then K_i/N_i is strictly decreasing in i, and L_i/N_i and $(K_i(1-\beta) + L_i)/N_i$ are strictly increasing in i.

<u>Proof.</u>

$$K_i/N_i - K_{i-1}/N_{i-1} = \frac{K_i\,L_{i-1} - K_{i-1}\,L_i}{(K_i+L_i)\,(K_{i-1}+L_{i-1})}$$

so K_i/N_i is increasing, constant, or decreasing according to whether $K_i\,L_{i-1} - K_{i-1}\,L_i$ is positive, zero, or negative,

$$
K_i\,L_{i-1} - K_{i-1}\,L_i
$$

$$
= (1-\alpha_1)(1-\alpha_0)^{i-2} \left\{ \alpha_1(1-\beta)^{i-1} + \alpha_0(1-\alpha_1)\,(1-\beta)^{i-2} \sum_{k=0}^{i-2} \left(\frac{1-\alpha_0}{1-\beta}\right)^k \right.
$$

$$
\left. - (1-\alpha_0)\left[\alpha_1(1-\beta)^{i-2} + \alpha_0(1-\alpha_1)\,(1-\beta)^{i-3} \sum_{k=0}^{i-3} \left(\frac{1-\alpha_0}{1-\beta}\right)^k \right] \right\}
$$

$$
= (1-\alpha_1)(1-\alpha_0)^{i-2}\,(1-\beta)^{i-2} \left\{ \alpha_1(1-\beta) + \alpha_0(1-\alpha_1) \sum_{k=0}^{i-2} \left(\frac{1-\alpha_0}{1-\beta}\right)^k \right.
$$

$$
\left. - \alpha_1(1-\alpha_0) - \alpha_0(1-\alpha_1)\,\frac{1-\alpha_0}{1-\beta} \sum_{k=0}^{i-3} \left(\frac{1-\alpha_0}{1-\beta}\right)^k \right\}
$$

$$
= (1-\alpha_1)(1-\alpha_0)^{i-2}\,(1-\beta)^{i-2}\,\{\alpha_0 - \alpha_1\beta\} \quad .
$$

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This proves the assertions regarding K_i/N_i. The other assertions follow directly from the formulas

$$L_i/N_i = 1 - K_i/N_i \ ,$$

$$(K_i(1-\beta) + L_i)/N_i = 1 - \beta K_i/N_i \ . \qquad \square$$

Using the above proposition, we can now compare $V(s,n)$ and $V(s)$ for various values of s.

Theorem 1.

i) If $\alpha_0 \geq \alpha_1 \beta$, then $V(s,n)$ and $V(s)$ are non-increasing in $s \geq 1$.

ii) If $\alpha_0 \leq \alpha_1 \beta$, then $V(s,n)$ and $V(s)$ are non-decreasing in $s \geq 1$.

Proof. (i) For $n = 1$, $V(s,n)$ is constant for $s \geq 0$ and therefore non-increasing. Now using induction assume that $V(s, n-1)$ is non-increasing in $s \geq 1$. The two arguments of the maximization operator in the recursive expression for $V(s,n)$ (Proposition 3) are

$$K_s(1-\beta) \ V(0, \ n-1)/N_s + L_s \ V(1, \ n-1)/N_s$$

$$= (1-\beta) \ V(0, \ n-1) + L_s(V(1, \ n-1) - (1-\beta) \ V(0,n-1))/N_s, \qquad (3)$$

and

$$(K_s(1-\beta) + L_s) \ V(s+1, \ n-1)/N_s \ . \qquad (4)$$

By Proposition 4, $(K_s(1-\beta) + L_s)/N_s$ is decreasing in s and by hypothesis $V(s, n-1)$ is non-increasing in s. Therefore (4) is non-increasing. Again by Proposition 4, L_s/N_s is non-increasing and by Lemma 2 which follows $V(1, n-1) - (1-\beta) V(0, n-1) \geq 0$. Thus (3) is also non-increasing and so $V(s, n)$ is non-increasing in $s \geq 1$. Since $V(s) = \lim_{n \to \infty} V(s,n)$, $V(s)$ must be non-increasing in $s \geq 1$ also.

The proof of part (ii) is identical except for the use of part (iii) of Proposition 4 instead of part (i). $\square$

Lemma 1.

i) $V(0,n) = 1 + \delta(1-\gamma) V(0, n-1)$, $\qquad\qquad n \geq 0$.

ii) $V(0) = 1 + \delta(1-\gamma) V(0)$.

Proof. These results follow immediately from Proposition 3. $\square$

Lemma 2.

$$V(1,n) \geq (1-\beta) V(0,n) , \qquad n \geq 0 .$$

Proof. The proof is by induction. For $n = 0,1$ the result is clearly true. Now assume the result holds for $n-1$. By Proposition 3

$$V(1,n) \geq 1 + \delta(K_1(1-\beta) V(0,n-1)/N_1 + L_1 V(1, n-1)/N_1)$$

$$\geq 1 + \delta(K_1(1-\beta) V(0, n-1)/N_1 + L_1(1-\beta) V(0, n-1)/N_1)$$

$$\geq 1 + \delta(1-\beta) V(0, n-1)$$

$$\geq (1-\beta) [1 + \delta(1-\gamma) V(0, n-1)] .$$

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Thus by Lemma 1,

$$V(1,n) \geq (1-\beta) \; V(0,n) \quad .$$

Theorem 2.

i) If $\gamma \leq \alpha_1 \beta$ and $\alpha_0 \geq \alpha_1 \beta$, then $V(0,n) \geq V(1,n)$ and $V(0) \geq V(1)$.

ii) If $\gamma \geq \alpha_1 \beta$, then $V(0,n) \leq V(1,n)$ and $V(0) \leq V(1)$.

Proof. (i) For $n = 1$ the result is trivial. Now assume $V(0,n-1) \geq V(1, n-1)$. Since $\alpha_0 \geq \alpha_1 \beta$, $V(s, n-1)$ is non-increasing in $s \geq 1$ by Theorem 1, so via Proposition 3

$$V(1,n) \leq 1 + \delta(K_1(1-\beta) + L_1) \; V(0, n-1)/N_1$$

$$\leq 1 + \delta(1-\alpha_1\beta) \; V(0, n-1)$$

$$\leq 1 + \delta(1-\gamma) \; V(0, n-1) \quad .$$

Thus by Lemma 1, $V(1,n) \leq V(0,n)$.

(ii) Clearly $V(0,n) \leq V(1, n)$ holds for $n = 1$. Now assume that $V(0, n-1) \leq V(1, n-1)$. By Proposition 3 and Lemma 1,

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$$V(1,n) \geq 1 + \delta(K_1(1-\beta) + L_1) \; V(0, \; n-1)$$

$$\geq 1 + \delta(1 - \alpha_1\beta) \; V(0, \; n-1)$$

$$\geq 1 + \delta(1 - \gamma) \; V(0, \; n-1) = V(0,n) \; .$$

This completes the proof for the finite-horizon optimal values. The results regarding the infinite-horizon optimal values follow from equation (1). $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ □

In the case omitted by Theorem 2, $\alpha < \alpha_1\beta$ and $\alpha_0 < \alpha_1\beta$, it can be shown that neither the inequality $V(0,n) \geq V(1,n)$ nor the inequality $V(0,n) \leq V(1,n)$ holds in general.

4. <u>Determination of Optimal Inspection Policies</u>

In this section we determine the form of the optimal inspection policies for various ranges of the parameters.

<u>Theorem 3</u>. i) If $\alpha_0 \geq \alpha_1\beta$ and $\gamma \leq \alpha_1\beta$, then the optimal inspection policy for both the finite and infinite horizon problems is to inspect every period.

(ii) If $\alpha_0 \leq \alpha_1\beta$ and $\gamma \geq \alpha_1\beta$, then the optimal inspection policy for both the finite and infinite horizon problems is to never inspect.

Proof. The optimal action when the device is in state s facing an n period horizon is that action which achieves the maximum in Proposition 3.A.iii. Let

$$D(s,n) = K_s(1-\beta)\ V(0,\ n-1/N_s + L_s\ V(1,\ n-1)/N_s$$

$$- (K_s(1-\beta) + L_s)\ V(s+1,\ n-1)/N_s\ .$$

Then when the device is in state s facing a horizon of n periods it is optimal to inspect if $D(s,n) \geq 0$ and not to inspect if $D(s,n) \leq 0$. When $\alpha_0 \geq \alpha_1\beta$ and $\gamma \leq \alpha_1\beta$, $V(s,n)$ is non-increasing in $s \geq 0$ for all n by Theorems 1 and 2. Thus $D(s,n) \geq 0$ for all $s,n \geq 1$ and it is optimal to inspect. When $\alpha_0 \leq \alpha_1\beta$ and $\gamma \geq \alpha_1\beta$, $V(s,n)$ is non-decreasing in $s \geq 0$ for all n by Theorems 1 and 2. Thus $D(s,n) \leq 0$ for all $s,n \geq 1$ and the optimal policy is not to inspect. This completes the finite horizon proof.

The optimal action when the device is in state s facing an infinite horizon is the action corresponding to the argument which achieves the maximum in Proposition 3.B.ii (see [9]). To be more specific, let

$$D(s) = K_s(1-\beta)\ V(0)/N_s + L_s\ V(1)/N_s$$

$$- (K_s(1-\beta) + L_s)\ V(s+1)/N_s\ .$$

Then when in state s it is optimal to inspect if $D(s) \geq 0$ and not to inspect if $D(s) \leq 0$. The remainder of the proof is identical to the finite horizon case. □

The lemma which follows provides the basis for determining the form of optimal inspection policy when $\alpha_0 > \alpha_1\beta$ and $\gamma > \alpha_1\beta$.

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Lemma 3. If $\alpha_0 > \alpha_1\beta$ and $\gamma > \alpha_1\beta$, then for all $n = 1, 2, \ldots$

    i) $D(s,n)$ and $D(s)$ are non-decreasing in s,

    ii) $D(s+1, n-1) N_{s+1} - (1-\alpha_0) D(s, n-1) N_s \geq 0$, $(n > 1)$,

iii) $\alpha_0(1-\beta) V(0, n-1) + (1-\alpha_0) V(1, n-1) - (1-\alpha_1\beta) V(2, n-1) \geq 0$.

Proof. The proof is by induction on n. The inequalities (ii) and (iii) are included only because they are necessary in the proof of part (i). Let

$$G(n) = \alpha_0(1-\beta) V(0, n-1) + (1-\alpha_0) V(1, n-1) - (1-\alpha_1\beta) V(2, n-1) ,$$

$$n = 1, 2, \ldots ,$$

and

$$H(s,n) = D(s+1, n-1) N_{s+1} - (1-\alpha_0) D(s, n-1) N_s .$$

It is easy to verify that $D(s,1) = D(s,2) = 0$ for $s = 1, 2, \ldots$. Also, $G(1) = 0$ and $G(2) = (\alpha_1-\alpha_0)\beta \geq 0$. Finally, note that $H(s,2) = 0$ [$H(s,1)$ is undefined]. Thus the finite-horizon portion of the lemma holds for $n = 1, 2$. Now assume the lemma holds for $n = N-1$, where $N \geq 3$. Let

$$s^*(n) = \min\{s \geq 1 : D(s,n) \geq 0\}$$

where $s^*(n) = +\infty$ if $D(s,n) < 0$ for all s. Since $D(s, N-1)$ is assumed to be non-decreasing in s, $D(s, N-1)$ is non-negative if and only if $s \geq s^*(N-1)$. Thus

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$$V(s,\ N-1) = 1 + \delta(K_s(1-\beta) + L_s)\ V(s+1,\ N-2)/N_s\ ,$$

$$\text{for}\quad s < s*(N-1)\ ,\qquad\qquad (5)$$

and

$$V(s,N-1) = 1 + \delta K_s(1-\beta)\ V(0,\ N-2)/N_s$$

$$+ \delta L_s\ V(1,\ N-2)/N_s\ ,\quad \text{for}\quad s \geq s*(N-1)\ .$$

In the remainder of the proof we will make use of the following easily verified formulas for $K_s, L_s,$ and N_s without mention:

$$K_{s+1} = (1-\beta)K_s + \alpha_0 L_s\ ,$$

$$L_{s+1} = (1-\alpha_0)L_s\ ,\qquad\qquad (6)$$

$$N_{s+1} = (1-\beta)K_s + L_s\ .$$

<u>Case 1</u>: $s*(N-1) = 1$: In this case, by equations (5) and Lemma 1

$$D(s,N) = \Big\{K_s(1-\beta)\ [1 + \delta(1-\gamma)\ V(0,\ N-2)]$$

$$+ L_s[1 + \delta K_1(1-\beta)\ V(0,\ N-2)/N_1 + \delta L_1\ V(1,\ N-2)/N_1]$$

$$- (K_s(1-\beta) + L_s)\ [1 + \delta K_{s+1}(1-\beta)\ V(0,\ N-2)/N_{s+1}$$

$$+ \delta L_{s+1}\ V(1,\ N-2)/N_{s+1}]\Big\}\ /N_s\ ,$$

$$= \delta(1-\beta)\ (\beta-\gamma)\ V(0,\ N-2)\ K_s/N_s$$

$$- \delta(\alpha_1-\alpha_0)\ [V(1,N-2) - (1-\beta)\ V(0,N-2)]L_s/N_s\ .\qquad (7)$$

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Recalling Lemma 2, Proposition 4 and the assumptions that $\beta > \gamma$ and $\alpha_1 > \alpha_0$ we see that $D(s,N)$ is non-decreasing in s.

Case 2: $s*(N-1) > 1$: In this case we first show that $D(s,N)$ is non-decreasing in the ranges $[1, s*(N-1) - 1)$ and $[s*(N-1) - 1, \infty)$.

$1 \leq s < s*(N-1) - 1$: In this range of s, by equations (5) and Lemma 1

$$D(s,N) = \left\{ K_s(1-\beta) \, [1 + \delta(1-\gamma) \, V(0, N-2)] + L_s[1 + \delta(K_1(1-\beta)+L_1) \, V(2,N-2)/N_1] \right.$$

$$\left. - (K_s(1-\beta) + L_s) \, [1 + \delta(K_{s+1}(1-\beta) + L_{s+1}) \, V(s+2, N-2)/N_{s+1}] \right\}/N_s$$

$$= \delta(1-\gamma)(1-\beta) \, V(0,N-2)K_s/N_s + \delta(1-\alpha_1\beta) \, V(2, N-2) \, L_s/N_s$$

$$- \delta V(s+2, N-2) \, N_{s+2}/N_s$$

$$= \delta D(s+1, N-1) \, N_{s+1}/N_s - \delta[(1-\beta) \, V(0, N-2) \, K_{s+1}/N_3 + V(1,N-2)L_{s+1}/N_s]$$

$$+ \delta(1-\gamma)(1-\beta) \, V(0,N-2) \, K_s/N_s + \delta(1-\alpha_1\beta) \, V(2, N-2) \, L_s/N_s$$

$$= \delta D(s+1, N-1)N_{s+1}/N_s + \delta(1-\beta)(\beta-\gamma) \, V(0, N-2) \, K_s/N_s$$

$$- \delta G(N-1) \, L_s/N_s \ . \tag{8}$$

Since $N_{s+1}/N_s = 1 - \beta K_s/N_s$, this quantity is decreasing when $\alpha_0 > \alpha_1\beta$ and $\gamma > \alpha_1\beta$. Also, $D(s+1, N-1)$ is non-decreasing by the inductive

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assumption and in the range $s \in [1, \ s*(N-1) \ - \ 1)$ it is negative. Thus $\delta D(s+1, \ N-1)N_{s+1}/N_s$ is non-decreasing. Since by the inductive assumption $G(N-1) \geq 0$, the other two terms in the above expression for $D(s,N)$ are also non-decreasing, $D(s,N)$ is non-decreasing in this range of s.

$s*(N-1) \ - \ 1 \leq s < \infty$: In this range of s, by equations (5) and Lemma 1

$$D(s,N) = \Big\{ K_s(1-\beta) \ [1 + \delta(1-\gamma) \ V(0, \ N-2)]$$

$$+ \ L_s[1 + \delta(K_1(1-\beta) + L_1) \ V(2, \ N-2)/N_1]$$

$$- \ (K_s(1-\beta) + L_s) \ [1 + \delta K_{s+1}(1-\beta) \ V(0, \ N-2)/N_{s+1}$$

$$+ \ \delta L_{s+1} \ V(1, \ N-2)/N_{s+1}] \Big\} \ /N_s \ ,$$

$$= \delta(1-\gamma) \ (1-\beta) \ V(0, \ N-2)K_s/N_s + \delta(1 - \alpha_1\beta) \ V(2, \ N-2)L_s/N_s$$

$$- \ \delta(1-\beta) \ V(0, \ N-2)K_{s+1}/N_s - \delta V(1, \ N-2)L_{s+1}/N_s \ ,$$

$$= \delta(1-\beta)(\beta-\gamma) \ V(0, \ N-2)K_s/N_s - \delta G(N-1)L_s/N_s \ . \qquad (9)$$

Again by the inductive assumption $G(N-1) \geq 0$, so $D(s,N)$ is non-decreasing in this range of s. To show that $D(s,N)$ is non-decreasing throughout $[1,\infty)$ all that remains to prove is that $D(s*(N-1) \ - \ 1, \ N) \ - \ D(s*(N-1)-2,N) \geq 0$. Using equations (8) and (9) and denoting $s*(N-1)$ for the time being simply by $s*$,

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$D(s*-1, N) - D(s*-2, N)$

$$= \delta(1-\beta)(\beta-\gamma) \; V(0, \; N-2)K_{s*-1}/N_{s*-1} - \delta G(N-1)L_{s*-1}/N_{s*-1}$$

$$- \delta D(s*-1, \; N-1)N_{s*-1}/N_{s*-2} - \delta(1-\beta)(\beta-\gamma) \; V(0, \; N-2)K_{s*-2}/N_{s*-2}$$

$$+ \delta G(N-1)L_{s*-2}/N_{s*-2}$$

$$= \delta(1-\beta)(\beta-\gamma) \; V(0, \; N-2) \; [K_{s*-1}/N_{s*-1} - K_{s*-2}/N_{s*-2}]$$

$$+ \delta G(N-1) \; [L_{s*-2}/N_{s*-2} - L_{s*-1}/N_{s*-1}] - \delta D(s*-1, \; N-1)N_{s*-1}/N_{s*-2} \; .$$

Since K_s/N_s is increasing and L_s/N_s is decreasing the first two terms in the above expression are non-negative. Also, by the definition of $s* = s*(N-1)$, $D(s*-1, \; N-1) < 0$. Thus $D(s*-1, \; N) - D(s*-2, \; N) \geq 0$. Next we show that $H(s,N) \geq 0$ for all s.

Case 1: $s*(N-2) = 1$: Using equation (7) and simplifying,

$$H(s,N) = \delta(1-\beta)(\beta-\gamma) \; V(0, \; N-3) \; [K_{s+1} - (1-\alpha_0)K_s] \geq 0$$

by Lemma 4 (which follows this proof).

Case 2: $s*(N-2) > 1$, $s \geq s*(N-2) - 1$: In this case, using equation (9) and simplifying,

$$H(s,N) = \delta(1-\beta)(\beta-\gamma) \; V(0, \; N-3) \; [K_{s+1} - (1-\alpha_0)K_s] \geq 0$$

by Lemma 4.

<u>Case 3</u>: $s*(N-2) > 1$, $s+1 = s*(N-2) - 1$: Using equations (8) and (9) and simplifying

$$H(s,N) = \delta(1-\beta)(\beta-\gamma) \; V(0, N-3) \; [K_{s+1} - (1-\alpha_0)K_s]$$

$$- \delta(1-\alpha_0) \; D(s+1, N-2) \; N_{s+1} \quad .$$

The first term in the above is non-negative by Lemma 4. The second term is non-positive because $s+1 < s*(N-2)$ and so $D(s+1, N-2) < 0$.

<u>Case 4</u>: $s+1 < s*(N-2) - 1$: In this case, using equation (8) and simplifying,

$$H(s,N) = \delta(1-\beta)(\beta-\gamma) \; V(0, N-3) \; [K_{s+1} - (1-\alpha_0)K_s] + \delta H(s+1, N-1) \quad .$$

Again the first term is non-negative by Lemma 4. The second term is non-negative by the inductive hypothesis. Hence we have shown that $H(s,N) \geq 0$ for all s.

Next we show that $G(N) \geq 0$.

<u>Case 1</u>: $s*(N-1) \leq 2$: By Proposition 3.A.iii,

$$V(1, N-1) \geq 1 + \delta K_1(1-\beta) \; V(0, N-2)/N_1 + \delta L_1 \; V(1, N-2)/N_1 \quad .$$

Combining this inequality with the expressions for $V(0, N-2)$ and $V(2, N-2)$ given by Lemma 1 and equations (5),

$$G(N) \geq \alpha_0(1-\beta) \ [1 + \delta(1-\gamma) \ V(0, \ N-2)]$$

$$+ (1-\alpha_0) \ [1 + \delta\alpha_1(1-\beta) \ V(0, \ N-2) + \delta(1-\alpha_1) \ V(1, \ N-2)] \quad (10)$$

$$- (1-\alpha_1\beta) \ [1 + \delta K_2(1-\beta) \ V(0, \ N-2)/N_2 + \delta L_2 \ V(1, \ N-2)/N_2]$$

$$\geq (\alpha_1-\alpha_0)\beta + \delta V(0, \ N-2) \ [\alpha_0(1-\beta) \ (1-\gamma) + \alpha_1(1-\beta)(1-\alpha_0) - K_2(1-\beta)]$$

$$+ \delta V(1, \ N-2) \ [(1-\alpha_0) \ (1-\alpha_1) - L_2]$$

$$\geq (\alpha_1-\alpha_0)\beta + \delta(1-\beta) \ [\alpha_1\beta - \gamma\alpha_0] \ V(0, \ N-2) \quad . \qquad (11)$$

Since $\alpha_1 > \alpha_0 > 0$ and $\beta > \gamma > 0$, $G(N) \geq 0$.

Case 2: $s*(N-1) > 2$. In this case

$$G(N) = \alpha_0(1-\beta) \ [1 + \delta(1-\gamma) \ V(0, \ N-2)]$$

$$+ (1-\alpha_0) \ [1 + \delta(K_1(1-\beta) + L_1) \ V(2, \ N-2)/N_1]$$

$$- (1-\alpha_1\beta) \ [1 + \delta(K_2(1-\beta) + L_2) \ V(3, \ N-2)/N_2]$$

$$= \alpha_0(1-\beta) \ [1 + \delta(1-\gamma) \ V(0, \ N-2)]$$

$$+ (1-\alpha_0) \ [1 + \delta\alpha_1(1-\beta) \ V(0, \ N-2) + \delta(1-\alpha_1) \ V(1, \ N-2)]$$

$$- (1-\alpha_1\beta) \ [1 + \delta K_2(1-\beta) \ V(0, \ N-2)/N_2 + \delta L_2 \ V(1, \ N-2)/N_2]$$

$$+ (1-\alpha_0) \ [1 + \delta N_2 \ V(2, \ N-2) - 1$$

$$- \delta\alpha_1(1-\beta) \ V(0, \ N-2) - \delta(1-\alpha_1) \ V(1, \ N-2)]$$

$$- (1 - \alpha_1\beta) \ [1 + \delta N_3 \ V(3, \ N-2)/N_2 - 1$$

$$- \delta K_2(1-\beta) \ V(0, \ N-2)/N_2 - \delta L_2 \ V(1, N-2)/N_2] \quad .$$

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Now the first three terms in the above are the same as those in equation (10), which simplifies to (11). Therefore

$$G(N) \geq (\alpha_1 - \alpha_0)\beta + \delta(1-\beta) [\alpha_1\beta - \gamma\alpha_0] V(0, N-2) + \delta H(1,N) .$$

Since we have proved above that $H(1,N) \geq 0$, $G(N) \geq 0$.

It remains only to show that $D(s)$ is non-decreasing. By equation (1)

$$D(s) = \lim_{n \to \infty} D(s,n) .$$

Since each $D(s,n)$ is non-decreasing in s, $D(s)$ must be non-decreasing also. □

Lemma 4. If $\alpha_0 > \alpha_1\beta$ then $K_{s+1} \geq (1-\alpha_0)K_s$.

Proof. Case 1: $\alpha_0 \geq \beta$: Here

$$K_{s+1} - (1-\alpha_0)K_s = (\alpha_0-\beta)K_s + \alpha_0 L_s \geq 0 .$$

Case 2: $\alpha_0 < \beta$: $K_{s+1} - (1-\alpha_0)K_s = \alpha_0 N_s - \beta K_s = N_s[\alpha_0 - \beta K_s/N_s]$. The sign of the above expression is determined by that of $\alpha_0 - \beta K_s/N_s$ and since this quantity is decreasing all we have to show is that

$$\lim_{s \to \infty} K_s/N_s \leq \alpha_0/\beta .$$

From Proposition 2,

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$$K_s / N_s = \frac{(\alpha_0 - \beta) \, \alpha_1 (\frac{1-\beta}{1-\alpha_0})^{s-1} + \alpha_0 (1-\alpha_1) \, [(\frac{1-\beta}{1-\alpha_0})^{s-1} - 1]}{(\alpha_0 - \beta) \, \alpha_1 (\frac{1-\beta}{1-\alpha_0})^{s-1} + \alpha_0 (1-\alpha_1) \, [(\frac{1-\beta}{1-\alpha_0})^{s-1} - 1] + (1-\alpha_1)(\alpha_0 - \beta)} \; .$$

Thus

$$\lim_{s \to \infty} K_s / N_s = \alpha_0 / \beta \; .$$

<u>Theorem 4</u>. If $\alpha_0 > \alpha_1 \beta$ and $\gamma > \alpha_1 \beta$ then

i) if the device is in state s facing an n-period horizon, it is

optimal to inspect if and only if $s \geq s*(n)$,

ii) if the device is in state s facing an infinite horizon, it is

optimal to inspect if and only if $s \geq s*(\infty) = \min\{s : D(s) \geq 0\}$.

<u>Proof</u>. This theorem is an immediate consequence of Lemma 3. □

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The form of the optimal inspection policy has been analyzed in every case except $\alpha_0 < \alpha_1 \beta$ and $\gamma < \alpha_1 \beta$. In this case it turns out that $D(s,n)$ crosses zero at most once and from above (s varying, n fixed) and so it is optimal to inspect in state s unless s is larger than some number $z*(n)$.

Lemma 5. If $\alpha_0 < \alpha_1 \beta$ and $\gamma < \alpha_1 \beta$, then as s increases $D(s,n)$ and $D(s)$ cross zero at most once and from above.

Proof.

$$D(s,n) = [V(0,n-1) - V(s+1, n-1)](1-\beta)K_s/N_s$$

$$+ [V(1, n-1) - V(s+1, n-1)]L_s/N_s . \tag{12}$$

By Theorem 1.ii, $V(s+1, n-1)$ is non-decreasing in s and by Proposition 4.iii K_s/N_s is decreasing and L_s/N_s is increasing. Consider the interval of s for which $V(0, n-1) \geq V(s+1, n-1)$. In this range $V(0, n-1) - V(s+1, n-1)$ is non-negative and is non-increasing. Thus in this range the first term in (12) is non-increasing. Also $V(1, n-1) - V(s+1)$ is non-positive and non-increasing, so the second term is also non-increasing. Thus $D(s,n)$ is non-increasing for s in this range. For the interval of s for which $V(0, n-1) < V(s+1, n-1)$, both terms in (12) are negative and thus $D(s, n-1) < 0$. Thus $D(s,n)$ crosses zero at most once and from above, as desired. Since $D(s) = \lim_{n \to \infty} D(s,n)$ it is easy to verify that $D(s)$ must also have this property. □

Let

$$z*(n) = \max\{s: D(s,n) \geq 0\}$$

and

$$z*(\infty) = \max\{s: D(s) \geq 0\}$$

where we take the maximum over an empty set to be $-\infty$.

<u>Theorem 5</u>. If $\alpha_0 < \alpha_1\beta$ and $\gamma < \alpha_1\beta$ then if the device is in state s facing an n-period horizon, it is optimal to inspect if and only if $s \leq z*(n)$, $(1 \leq n \leq \infty)$.

<u>Proof</u>: Follows immediately from Lemma 5. □

5. <u>Conclusions</u>

The form of the optimal policy has been shown to depend in a simple way upon γ, α_0, and $\alpha_1\beta$. The quantities α_0 and γ have obvious interpretations; $\alpha_1\beta$ is the probability that given the device is currently in observed state 1 (i.e., last inspection one period ago; results OK) that the device will be failed at the start of the next period. When $\gamma \leq \alpha_1\beta$, observed state 0 seems intuitively to be a better state to be in than state 1, and conversely. The relation between α_0 and $\alpha_1\beta$ has been shown to determine whether or not the conditional probabilities K_s/N_s amd L_s/N_s are increasing or decreasing. With these facts noted, the forms of the optimal policy in the various cases are intuitively sensible.

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The optimal policy when $\alpha_0 < \alpha_1\beta$ and $\gamma < \alpha_1\beta$ perhaps deserves

some special discussion because of its seemingly odd nature. When the

device is in observed state s, inspection may seem intuitively

desirable since if the true state is 2, then in every period in

which the partial failure remains undetected there is a larger probability

of complete failure. When $\gamma > \alpha_1\beta$ this preference for inspection indeed

should hold for s = 1. However, when $\alpha_0 < \alpha_1\beta$, L_s/N_s is increasing,

so that as s increases the probability that the device is in true state

1 (OK) increases also. Of course when the true state is 1, inspection

is not desirable. Thus it seems reasonable that once the observed state s

is larger than some number, L_s/N_s is sufficiently large to make not

inspecting optimal. Finally we should note that when $\alpha_0 < \alpha_1\beta$ and

$\gamma < \alpha_1\beta$ the optimal policy for a device starting out in observed state

$s \leq z^*(\infty)$ will be to always inspect, and so the device will never in

the future reach any observed state greater than 1. However if the device

starts out in observed state $s > z^*(\infty)$, then the optimal policy will be to

never inspect.

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In this paper we consider a hazardous inspection model. One is given a device which operates throughout a number of periods and which in each period may be subject to failure. Failure of the device is directly observable. Prior to failure the device will enter a state in which it is functioning, but in an impaired manner. This state can be detected only by performing an inspection. Once the machine is known to be in the impaired state, appropriate action may be taken to prolong the remaining life of the device. In this respect inspection is valuable. However the act of inspecting the device when it is not impaired may itself cause the device to become impared. In this respect inspection is hazardous. This paper deals with the determination of inspection policies which maximize the expected lifetime of the device.

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